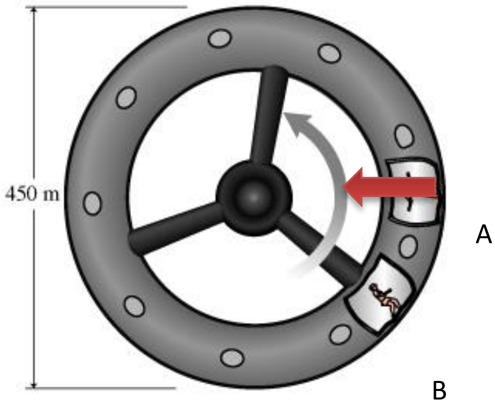
06 Circular motion and gravitation review questions

Part A Uniform Circular Motion.



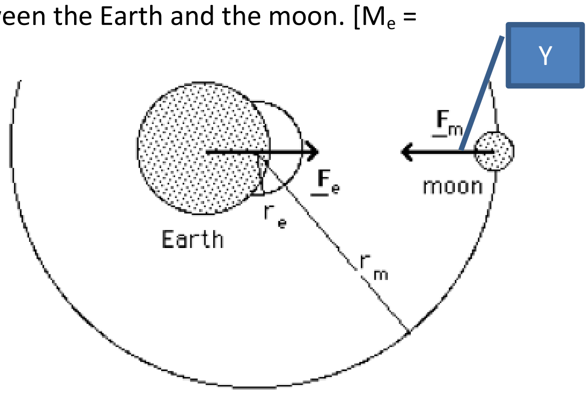
1. A circular shaped space station has a 450m diameter and rotates to create an acceleration that results in a gravity like effect at its perimeter for the occupants.
    a. Add an arrow to show the direction of the force on person A.
    b. State the magnitude and direction of the acceleration at the perimeter in order to produce a gravitational effect equivalent to 0.5g.
    Towards the centre: 0.5x9.81 = 4.91 ms^{-2}
    c. Calculate, for achieving this effect the….
        (i) Velocity at the edge
        $a = v^2/r$……so $v = \sqrt{(4.91 \times 225)} = 33.2 ms^{-1}$
        (ii) the angular velocity
        $v = \omega r$………..so $\omega = v/r = 33.2/225 = 0.148$ rads^{-1}.
        (iii) the time taken to create angular displacement of π radians
        angular displacement = angular velocity x time
        time = π / 0.148 = 21.3 s
        (or $a = 4\pi^2 r/T^2$ so $T = 2\pi\sqrt{(r/a)} = 2\pi\sqrt{(225/4.91)} = 42.5$ s
        Π radians is half a period so time taken = 42.5/2 = 21.3 s

Part B: Newton's Law of Gravitation

1. The gravitational field strength at the Earth's surface is g. g = 9.81 N/kg.
    a. How much does a 62.3kg person weigh on Earth?
    W = mg = 62.3 x 9.81 = 611 N
    b. The radius of the Earth is 6.371×10^6 meters. What is Earth's mass?
    $F = GMm/r^2$…………..$M = Fr^2/Gm$
    $M = 611 \times (6.371 \times 10^6)^3 / 6.67 \times 10^{-11} \times 62.3 = 5.969 \times 10^{24}$kg
2. At increased height above sea level reduces there is less gravitational field strength. At the top of Mount Everest the gravitational field strength is 9.78 N/kg.
    a. Estimate the height of Mount Everest.
    $g = GM/r^2$……….$r = \sqrt{(GM/g)} = \sqrt{(6.67 \times 10^{-11} \times 5.969 \times 10^{24}\text{kg}/9.78)} = 6.380 \times 10^6$m
    $6.380 \times 10^6 - 6.371 \times 10^6 = 9000$m
    b. Explain how the rotation of the Earth affects measurements of g.
    The acceleration due to gravity as measured will be less because part of the gravitational acceleration is used to accelerate all the objects centripetally. Thiss effect is greatest near the equator

3.

a. Calculate the average force of attraction between the Earth and the moon. [M_e = 6.0×10^{24} kg, M_m = 7.3×10^{22}kg, Average distance from the Earth to the Moon is 3.8×10^8m].



$F = GMm/r^2$

$F = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 7.3 \times 10^{22}/(3.8 \times 10^8)^2$

$= 2.02 \times 10^{20}$N

b. Calculate the gravitational field strength at the surface of the moon (radius 1740km).

$g = GM/r^2$

$g = 6.67 \times 10^{-11} \times 7.3 \times 10^{22}/(1.74 \times 10^6)^2 = 1.61$ N/kg

c. Calculate the gravitational field strength due to the moon and the Earth combined at a point Y. 3.8×10^7m from the moon in a direction directly towards the Earth.

$g = GM/r^2$ for the moon and then for the Earth. The overall field strength is these two subtracted because they are in opposite directions. Overall field strength should be very small the attraction of the Earth and Moon cancel at this point: (NB it is 3.8×10^8m - 3.8×10^7m = from the Earth)

Moon:

$g = 6.67 \times 10^{-11} \times 7.3 \times 10^{22}/(3.8 \times 10^7)^2 = 0.00337$ N/kg

Earth:

$g = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} /(3.42 \times 10^8)^2 = 0.00342$ N/kg

so resultant $g = 0.00342 - 0.00337 = 1.5 \times 10^{-6}$N/kg (i.e. very small)